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Neutrino Anomalies in an Extended Zee Model

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Abstract

We discuss an extended $SU(2) \times U(1)$ model which naturally leads to mass scales and mixing angles relevant for understanding both the solar and atmospheric neutrino anomalies. No right-handed neutrinos are introduced in the model. The model uses a softly broken $L_e - L_\mu - L_\tau$ symmetry. Neutrino masses arise only at the loop level. The one-loop neutrino masses which arise as in the Zee model solve the atmospheric neutrino anomaly while breaking of $L_e - L_\mu - L_\tau$ generates at two-loop order a mass splitting needed for the vacuum solution of the solar neutrino problem. A somewhat different model is possible which accommodates the large-angle MSW resolution of the solar neutrino problem.

Recent results on atmospheric neutrinos from SuperKamiokande [1] seem to supply definite evidence that muon neutrinos oscillate into either tau neutrinos or some species of sterile neutrinos. The evidence that solar neutrinos (ν_e) oscillate into some other species has also been mounting. Simultaneous

understanding of these experiments needs two hierarchical mass scales Δ_A and Δ_S in the neutrino sector [2]. The standard picture with only three generations of light neutrinos can accommodate two mass scales. But theoretical understanding of the pattern of masses and mixings within conventional pictures of neutrino mass generation is not straightforward. Basically, apart from understanding why neutrino masses are small one would also need to have theoretical understanding of the (i) two hierarchical mass scales and (ii) presence of the one or two large mixing angles in the leptonic Kobayashi-Maskawa matrix. The requirements (i) and (ii) becomes particularly stringent in case of the vacuum oscillations (VO) solution [3] for the solar neutrino problem which needs $\frac{\Delta_S}{\Delta_A} \sim 10^{-8}$ and large mixing angle¹.

One of the possibilities for understanding the smallness of the neutrino mass is to assume that it arises radiatively². An attractive possibility is that Δ_A is small because it arises at one loop in perturbation theory and Δ_S is still smaller because it arises at the two-loop level. We describe here precisely such a scheme in the context of an $SU(2)_L \times U(1)$ electroweak gauge theory.

The Zee model [8] provides a particularly nice example of radiative neutrino mass generation. It extends only the Higgs sector of the standard model (SM) in a way that admits lepton number violation but no neutrino masses at the tree level. In the most general situation, all three neutrinos obtain their masses at the one-loop level in this model. But the predicted spectrum and the mixing pattern is quite constrained. In spite of that, the Zee model has been argued recently [9] to provide a very good zeroth order approximation to the required neutrino spectrum for definite choice of parameters of the model. Specifically, it was found [9] that the only way in which one could understand the hierarchical mass scales and the required mixing pattern in this model is to augment it with a (global) $L_e - L_\mu - L_\tau$ symmetry³. This symmetry has been earlier recognized [4] to lead to one and possibly

¹Some of the models or textures leading to vacuum solution for the solar neutrino anomalies are presented in [4] and in [5]. More detailed list of references can be found in [2]

²A review and references for models with radiatively generated neutrino masses can be found in [6]. See also [7].

³Another possible interpretation of the Zee model is that it admits the dark matter and the atmospheric mass scales [10]. In this case it can explain the $\nu_\mu - \nu_\tau$ and possible $\nu_\mu - \nu_e$ oscillations seen at SuperKamiokande and LSND respectively. But it cannot accommodate a solution to the solar neutrino problem unless a singlet neutrino is invoked [10, 11].

two large mixing angles and two degenerate and one massless neutrinos. The muon neutrino deficit can be explained in the model by identifying the degenerate mass with the atmospheric scale. Moreover, the bimaximal mixing pattern [12] possible in this case provides an understanding of the absence of the electron neutrino oscillation at the atmospheric scale found at SuperKamiokande and at CHOOZ. The smallness of the atmospheric scale also becomes understandable in the Zee model due to its radiative origin.

While providing an understanding of all the features of the atmospheric neutrino results, the Zee model plus $L_e - L_\mu - L_\tau$ symmetry fails to lead to the required solar scale. This needs a small breaking of the $L_e - L_\mu - L_\tau$ symmetry. Our aim here is to discuss a specific extension of the Zee model in which the hierarchical $\frac{\Delta_s}{\Delta_A}$ gets generated at the two-loop level. The value of the $\frac{\Delta_s}{\Delta_A}$ obtained in the model would lead to vacuum oscillations as solution to the solar neutrino scale naturally but it is also possible to obtain the large-angle MSW (LMSW) [13] solution in this type of picture.

The model described below is in some sense a combination of the models of Zee [8] and Babu [14], with the added feature that the Yukawa couplings conserve total lepton number L , as well as $L' \equiv L_e - L_\mu - L_\tau$.⁴

The model is based on the gauge group $SU(2)_L \times U(1) \times SU(3)_C$, and has the same fermion content as the standard model (SM). Thus, no singlet right-handed neutrinos are introduced. We introduce, in addition to the SM $SU(2)_L$ doublet scalar field ϕ_1 , another doublet ϕ_2 , and $SU(2)_L$ singlets h^+ and k^{++} which carry respectively charges $+1$ and $+2$. In addition to the total lepton number $L \equiv L_e + L_\mu + L_\tau$, an additional global symmetry corresponding to $L' \equiv L_e - L_\mu - L_\tau$ is introduced. The quantum numbers carried by scalar fields are shown in Table 1.

The Yukawa couplings of the leptons consistent with all symmetries are

$$\begin{aligned}
-\mathcal{L}_Y = & g_{ij}^{(a)} \overline{L}_{iL} \phi_a e_{jR} + f_{e\mu} (\overline{e}_R^c \nu_{\mu L} - \overline{\mu}_R^c \nu_{eL}) h^+ \\
& + f_{e\tau} (\overline{e}_R^c \nu_{\tau L} - \overline{\tau}_R^c \nu_{eL}) h^+ + h_{ee} \overline{e}_R^c e_L k^{++} + \text{H.c.}, \quad (1)
\end{aligned}$$

where $g_{ij}^{(a)}$ ($a = 1, 2 ; i, j = 1, 2, 3$) have vanishing (12), (13), (21) and (31) elements. Vacuum expectation values of ϕ_a generate the charged-lepton mass matrix, which gives masses to all charged leptons. While mixing between the

⁴Similar models have been discussed [15] in the past in the context of a 17 keV neutrino or hot dark matter. All these models invoked a sterile neutrino in addition to the three light neutrinos.

	$SU(2)_L$	Y	L	L'
$\phi_{1,2}$	$\underline{2}$	$-\frac{1}{2}$	0	0
h^+	$\underline{1}$	+1	-2	0
k^{++}	$\underline{1}$	+2	-2	-2

Table 1: Quantum numbers of the scalar fields under the various groups

second and third generations is permitted in this sector, there is no mixing between the first and second or third generations at tree level. The neutrinos are massless at tree level.

Apart from the usual bilinear and quartic terms in the fields conserving L as well as L' , the scalar potential of the model contains the soft symmetry breaking trilinear terms

$$V_3(\phi_a, h, k) = \mu \left(\tilde{\phi}_1^\dagger \phi_2 h^+ + \phi_2^\dagger \tilde{\phi}_1 h^- \right) + \kappa \left(h^+ h^+ k^{--} + h^- h^- k^{++} \right), \quad (2)$$

where $\tilde{\phi}_1 = i\tau_2 \phi_1^*$ is the doublet conjugate to ϕ_1 . The μ term is characteristic of the Zee model [8] and violates L by ± 2 units, whereas the κ term is characteristic of Babu model [14] and violates L as well as L' by ± 2 units.

Neutrino masses are generated in the model through exchanges of the physical charged Higgs bosons of the model. A linear combination of ϕ_1^\pm and ϕ_2^\pm is eaten up by W^\pm while the orthogonal combination ($\equiv \phi^\pm$) mixes with the h^\pm through the μ term. This magnitude of this mixing is quite small, $\approx \mu v / m_h^2$, for $v \equiv \sqrt{\langle \phi_1^0 \rangle^2 + \langle \phi_2^0 \rangle^2} \ll m_h^2 / \mu$. We therefore continue to denote the physical charged Higgs bosons as h^\pm and ϕ^\pm .

At the one-loop level, the diagram in Fig. 1 contributes to the neutrino mass matrix, which takes the form

$$M_\nu^{(1)} = m_\nu \begin{pmatrix} 0 & \cos \theta_\nu & \sin \theta_\nu \\ \cos \theta_\nu & 0 & 0 \\ \sin \theta_\nu & 0 & 0 \end{pmatrix}, \quad (3)$$

which conserves $L' \equiv L_e - L_\mu - L_\tau$, while violating L by two units. Here [10]

$$m_\nu \approx \frac{\mu}{16\pi^2} \frac{\sqrt{f_{e\mu}^2 m_\mu^4 + f_{e\tau}^2 m_\tau^4}}{m_h^2} \log \frac{m_h^2}{m_\phi^2}, \quad (4)$$

and

$$\tan \theta_\nu \approx \frac{f_{e\tau} m_\tau^2}{f_{e\mu} m_\mu^2}. \quad (5)$$

In deriving the above relations it is assumed that one of the Yukawa couplings ($g^{(1)}$) makes the dominant contribution, and that $\langle \phi_1^0 \rangle \approx \langle \phi_2^0 \rangle$. Relaxing some of these simplifying assumptions would only give rise to more free parameters, making it easier to fit the data.

The mass matrix M_ν has eigenvalues $\pm m_\nu, 0$. At this stage the spectrum consists of a massless neutrino and a Dirac neutrino. For m_ν to provide the scale of atmospheric neutrino oscillations, i.e., for $m_\nu \approx 3 \cdot 10^{-2} - 10^{-1}$ eV, we require, assuming $m_h \approx 10^4$ GeV and $\mu \approx \langle \phi_a^0 \rangle$,

$$\sqrt{f_{e\mu}^2 m_\mu^4 + f_{e\tau}^2 m_\tau^4} \approx (0.4 - 1.2) \cdot 10^{-4} \text{ GeV}^2. \quad (6)$$

$M_\nu^{(1)}$ is diagonalized by a mixing matrix

$$U_\nu^{(1)} = \begin{pmatrix} 1/\sqrt{2} & 1/\sqrt{2} & 0 \\ \cos \theta_\nu / \sqrt{2} & -\cos \theta_\nu / \sqrt{2} & -\sin \theta_\nu \\ \sin \theta_\nu / \sqrt{2} & -\sin \theta_\nu / \sqrt{2} & \cos \theta_\nu \end{pmatrix}. \quad (7)$$

This matrix displays the bimaximal structure [12] if $\theta_\nu = 45^\circ$. This choice leads to the maximal amplitude for the ν_μ oscillations in conformity with the data on atmospheric neutrinos. Eq. (5) then implies

$$f_{e\tau} m_\tau^2 \approx f_{e\mu} m_\mu^2. \quad (8)$$

Eqs. (6, 8) together require

$$f_{e\mu} \approx 4 \cdot 10^{-1}, f_{e\tau} \approx 10^{-3}. \quad (9)$$

Having obtained the conditions for resolving the atmospheric neutrino anomaly we can now examine how a two-loop contribution to the neutrino mass matrix can provide a resolution of the solar neutrino problem.

The two-loop diagram contributing to the neutrino masses is shown in Fig. 2. Assuming that the k mass is dominant, the mass matrix at the two-loop level can be estimated to be

$$M_\nu^{(2)} = \begin{pmatrix} 0 & m_\nu \cos \theta_\nu & m_\nu \sin \theta_\nu \\ m_\nu \cos \theta_\nu & A f_{e\mu}^2 & A f_{e\mu} f_{e\tau} \\ m_\nu \sin \theta_\nu & A f_{e\mu} f_{e\tau} & A f_{e\tau}^2 \end{pmatrix}, \quad (10)$$

where

$$A \approx \frac{\kappa}{(16\pi^2)^2} h_{ee} \frac{m_e^2}{m_k^2}, \quad (11)$$

where we have assumed the logarithm factors to be of order unity. We see that now the mass matrix $M_\nu^{(2)}$ no longer respects the symmetry L' . Consequently, the two eigenvalues ($\pm m_\nu$) which were equal and opposite at one-loop level, are now split by an amount $A f_{e\mu}^2 \cos^2 \theta_\nu$. This leads to a mass-squared difference

$$\Delta_S \approx 2m_\nu A f_{e\mu}^2 \cos^2 \theta_\nu. \quad (12)$$

For $\kappa \approx 10^2$ GeV, $m_k \approx 10^4$ GeV, and $h_{ee} \approx 1$, this gives $\Delta_S \approx 5 \cdot 10^{-11}$ eV². This is in the range needed for the VO solution. It is of course obvious that the model does not permit a value of Δ_S in the range for the MSW solutions. We will discuss later the possibility of modifying the model to accommodate the LAMSW [13] solution.

We thus see that there is a choice of parameters $f_{e\mu}$, $f_{e\tau}$, h_{ee} , μ , κ , m_h and m_k for which simultaneous solutions to both atmospheric and solar neutrino anomalies can be found. Of these parameters, the masses m_h , m_k have to be somewhat large compared to the weak scale, and the ratio $f_{e\tau}/f_{e\mu}$ has to be somewhat fine tuned.

We now look at constraints on the model coming from other experiments. These have been studied in some detail in [8, 10, 14, 16]. However, many of these constraints are not relevant since many couplings vanish automatically because of the assumed L' invariance. Thus, for example, since k^{++} couples only to e^+e^+ , unlike in the Babu model [14], the only constraints on its mass and coupling are from $(g-2)$ of the electron and from the Bhabha scattering process $e^+e^- \rightarrow e^+e^-$. The former gives the limit [14]

$$|h_{ee}|^2/m_k^2 \lesssim 5 \cdot 10^{-2} \text{ GeV}^{-2}, \quad (13)$$

which is trivially satisfied for our choice of h_{ee} and m_k^2 . The limit from Bhabha scattering is [16]

$$|h_{ee}|^2/m_k^2 \lesssim 9.7 \cdot 10^{-6} \text{ GeV}^{-2}, \quad (14)$$

which is also satisfied.

So far as limits on the h^+ mass and couplings are concerned, these come from data on μ decay together with $e - \mu$ universality, and from $(g-2)$ of e

and μ . Of these, the former one is more stringent [14], viz.,

$$f_{e\mu}^2/m_h^2 \lesssim 10^{-8} \text{ GeV}^{-2}. \quad (15)$$

This constraint is satisfied for our choice of parameters.

It is straightforward to modify the above scheme so as to obtain the scale relevant for the MSW solution. The only change needed is in the quantum number of k^{++} from $L' = -2$ to $L' = +2$. The k^{++} Yukawa couplings in this case would be given by

$$-\mathcal{L}'_Y = [h_{\mu\mu}\overline{\mu}_R^c\mu_L + h_{\mu\tau}\overline{\mu}_R^c\tau_L + h_{\tau\mu}\overline{\tau}_R^c\mu_L + h_{\tau\tau}\overline{\tau}_R^c\tau_L] k^{++} + \text{H.c.} \quad (16)$$

Now the degenerate neutrinos at 1-loop level are split by the diagram of Fig.2 with the dominant contribution coming from diagram with internal τ lepton. This has the effect that under the simplifying assumptions of $h_{\mu\tau} = h_{\tau\mu} = 0$, and $h_{\mu\mu} \approx h_{\tau\tau}$, A defined in eq. (11) would be replaced by

$$A \approx \frac{\kappa}{(16\pi^2)^2} h_{\tau\tau} \frac{m_\tau^2}{m_k^2}. \quad (17)$$

Consequently, Δ_S would be larger compared to its value in eq.(12) due to m_τ replacing m_e in the expression for A . Hence,

$$\Delta_S \approx 8 \cdot 10^{-4} h_{\tau\tau} \text{ eV}^2. \quad (18)$$

This can be in the right range [13] for the LAMSW solution for $h_{\tau\tau} \sim 10^{-1} - 10^{-2}$.

The main obstacle in solving the solar neutrino problem in this case comes from the mixing pattern in eq.(7). This implies that the mixing angle determining the vacuum survival probability of the electron neutrino is exactly 45° and radiative corrections to it are too small to change it significantly. It is well-known [2, 10] that the MSW effect cannot take place inside the Sun in this case. A possible way out is to make a different L' assignment for ϕ_2 , viz., $L' = 2$. This has two consequences. Firstly, the charged lepton mass matrix no longer remains $L_e - L_\mu - L_\tau$ symmetric. Secondly, the $L_e - L_\mu - L_\tau$ breaking in the charged lepton would induce similar breaking in the neutrino mass matrix. Both these factor would lead to deviation from the bimaximal structure in eq.(7). It is possible to choose parameters g_{ij}^2 in a way which would make the vacuum amplitude $4U_{e1}^2 U_{e2}^2$ for the oscillations of the solar

neutrinos significantly less than 1 leading to a LAMSW solution to the solar neutrino problem.

To conclude, we have described an economical scenario which accommodates in a natural way the smallness of the neutrino masses, as well as the hierarchy in the scales Δ_S and Δ_A responsible for the understanding of the solar and atmospheric neutrino data. In the model we propose, the vacuum oscillation solution for the solar neutrino problem is natural. However, modified assignments of quantum numbers of scalars can give rise to a model which would accommodate the large-angle MSW solution.

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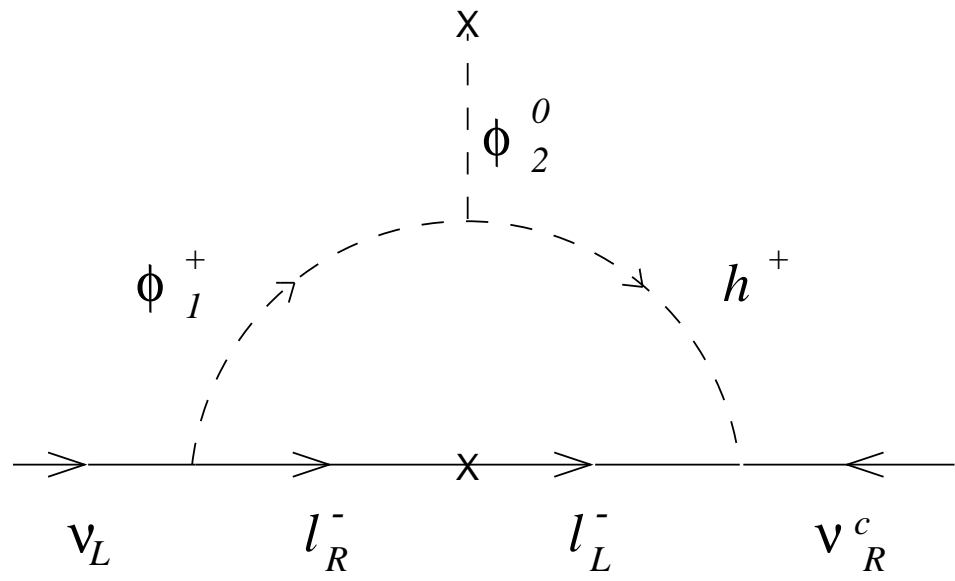


Figure 1: *One-loop diagram contributing to the neutrino mass matrix.*

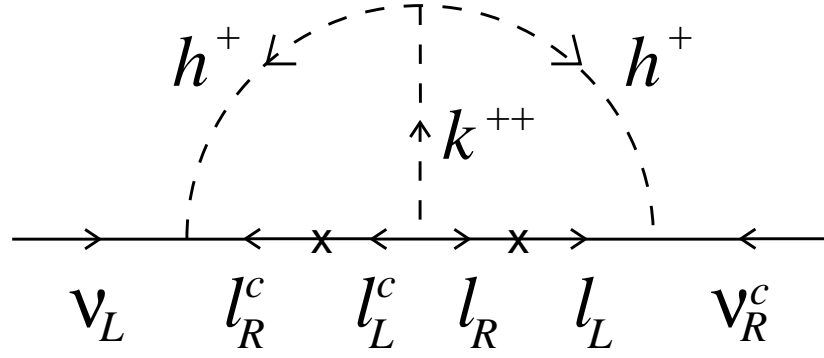


Figure 2: Two-loop diagram contributing to the neutrino mass matrix.